Chaotic Intermittency

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Chaotic intermittency is characterized by a signal that alternates aleatory between long regular (pseudo-laminar) phases and irregular bursts (pseudo-turbulent or chaotic phases).

chaotic intermittency types maps RPD noise

1. Introduction

Chaos theory has many fields of investigation, such as the different routes to chaos. One route is chaotic intermittency, which has been observed in several subjects. Batchelor and Townsend, in the middle of the previous century, used the word intermittency to give an account of the fluctuating velocity in turbulent flows [1]. Subsequently, intermittency has been found in many and varied physical phenomena such as the nonlinear behavior of transient periodic plasma and conducting fluids [2][3][4][5], fluid mechanics and turbulent flows [6][7][8][9][10], Rayleigh–Benard convection [11], electronic digital oscillator [12], logistic map [13], Alfven wave-fronts and derivative nonlinear Schrodinger equation [14][15], premixed combustion [16], Lorenz system [17], coupled oscillators [18][19][20][21], catalytic reactors [22], Ginzburg Landau equation [23], solar cycles [24], spatiotemporal chaos [25][26], thermoacoustic instability [27], control chaos [28], etc. Furthermore, chaotic intermittency is also observed in systems in economics [29][30], medicine, [31][32], neuroscience [33][34], genetics [35], and marine biology [36][37]. Therefore, a more suitable understanding of chaotic intermittency can collaborate to describe these phenomena accurately. In addition, the correct description of chaotic intermittency possesses significance for systems whose precise equations are partially or totally unknown.

There are three classical routes by which continuous or discrete dynamical systems can evolve from regular functioning to chaotic behaviors: quasi-periodic route, period-doubling scenario, and intermittency [38]. In chaotic intermittency, the dynamical system solutions display alternation between regular or pseudo-laminar phases and chaotic bursts or non-regular phases. The laminar phases correspond to pseudo-equilibrium, pseudo-periodic solutions, and quasi-invariant objects close to them that the system may consume for a long time. On the other hand, the burst ones are consistent with chaotic evolution [39].

Approximately 50 years ago, chaotic intermittency was categorized into three different types, known as I, II, and III [17][40][41]. This classification was according to the fixed point eigenvalues in the local Poincaré map or the Floquet multipliers of the continuous-system periodic solution [38][39][42]. Later works introduced other types of chaotic intermittency such as on–off, eyelet, ring, and in–out, type-X, and type-V [43][44][45][46][47][48].

The monodromy operator multipliers determine the type of intermittency [49]. Type-I intermittency happens if the periodic solution loses its stability by a cyclic-fold bifurcation [50], then a multiplier goes away from the unit circle by the real axis across +1. Type-II intermittency is born in a sub-critical Hopf bifurcation or a Neimark–Sacker bifurcation [38][51]. Accordingly, two complex-conjugate Floquet multipliers get out of the unit circle. Therefore, it is a consequence of a bifurcation scenario for T^2 torus breakdown. A sub-critical period-doubling bifurcation generates type-III intermittency. In this bifurcation, an unstable period-2 orbit encounters and destabilizes a stable period-1 orbit. Type-III intermittency presents a progressive increase throughout the laminar phase of a period-2 component in the motion [52]. In addition, a one-dimensional map $F(x)$ showing a sub-critical period-doubling bifurcation possesses a positive Schwartzian derivative, $SF(x)$:

$$SF\left(x\right) = F'''\left(x\right)/F'\left(x\right) - 1.5\left(F''\left(x\right)/F'\left(x\right)\right)^2 > 0\,. \qquad (1)$$

Chaotic intermittency may be investigated using one-dimensional maps (Poincaré maps) [38][39][42]. These maps have two main characteristics: a particular local map and a reinjection process or reinjection mechanism. The specific local map depicts the type of intermittency. The reinjection mechanism returns the trajectories from the chaotic regime to the laminar one. Therefore, the reinjection mechanism determines the reinjection probability density function (RPD). It considers only points in the laminar region, but in the preceding period they have not been there. Note that the RPD is used to describe the reinjection process and is defined by the chaotic dynamics of the system itself. The probability of the reinjection points being in a particular sub-interval inside the laminar interval equals the integral of the RPD in the sub-interval. The RPD function specifies the probability that trajectories are returned (reinjected) into the laminar zone around to the unstable or even the vanished fixed point, and together with the local map outline all the dynamical features of the system.

The precise determination of the RPD function is extremely significant in correctly describing the chaotic intermittency phenomenon. In addition, the evaluation of the RPD function from experimental or numerical is a hard task due to the large amount of data involved, and it is difficult to estimate the statistical fluctuations generated in the numerical result and experimental data. Several strategies were utilized to calculate the RPD function. Most results in the classical theory of chaotic intermittency were obtained considering uniform reinjection inside the laminar interval [41][42][53]. Other implemented RPD functions were constructed using specific characteristics of the nonlinear processes. For example, in type-I intermittency the reinjection was restricted to the fixed point [54], and for the intermittency of type-III the RPD function was assumed proportional to $(x{-}a)^{-1/2}$, with a= constant [55]. Notwithstanding, these RPD functions cannot be generalized to other nonlinear systems.

Two new methodologies together with their theoretical background to obtain the RPD function were introduced in recent years. One of them is the *M* function methodology [56][57][58][59][60][61][62][63][64]. This methodology introduces a generalized power law for the RPD. It has been shown to be very accurate for a broad class of one-dimensional maps showing type-I, II, III, and V intermittencies. In addition, the *M* function methodology includes the classical approximation because it contains uniform reinjection as a particular case. The second one is called the continuity technique. It utilizes the Perron–Frobenius operator to compute the reinjection probability density function. In the same way as the methodology of the *M* function, the continuity technique has been shown to be accurate for several maps displaying different types of intermittency [65][66][67].

To more precisely describe the intermittency phenomenon, other statistical functions are used, such as the probability density of the laminar lengths ($\psi(l)$), the average laminar length ($\bar{l}$), and the characteristic relation ($\bar{l} = \bar{l}(\varepsilon)$). Nonetheless, these functions depend on the RPD. To calculate them, researchers previously have to know the reinjection probability density function [39][42].

The RPD and the other statistical functions used to describe the chaotic intermittency are affected by the noise and the lower boundary of reinjection (LBR). The *M* function methodology has been extended to incorporate both phenomena [68][69][70].

2. Types of Chaotic Intermittency

Let reserachers analyze a periodic solution of an autonomous continuous-time system. It is stable for some values of the control parameters. At that time, if a control parameter is modified until the periodic solution loses stability, the development of the solution shall depend on how the Floquet multipliers go away from the unit circle in the complex plane [38][42][53]. Note that the metamorphoses of a family of solutions around a closed orbit is a complex problem of bifurcation theory [71].

The existence of type-I, II, and III intermittency depends on the monodromy operator multiplier. Type-I intermittency occurs by a cyclic-fold bifurcation, then a multiplier goes away from the unit circle across +1. Type-II intermittency is born in a sub-critical Hopf bifurcation (or a Neimark–Sacker bifurcation), in which two complex-conjugate multipliers move away from the unit circle. Finally, type-III intermittency appears if a multiplier gets out of the unit circle by −1, and a sub-critical period-doubling bifurcation happens [38][49][51].

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